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Students' Mathematical Representation and Communication Ability in Mathematics Problem Solving

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Abstract. *In learning mathematics, representation and communication ability are required by students to solve problems. The ability to represent is crucial for students to simplify the learning process, while students who have good mathematical communication abilities can easily solve a problem. This study applied a sequential mixed methods approach. Quantitative data was obtained from the results of the written test, then the ability of mathematical representation and communication in solving problems on linear program material was analyzed qualitatively. The participants of this study were 59 students from one of the senior high schools in Ambon, Indonesia. The research phase was begun with students being asked to solve mathematics problems and then researchers analyze representation and written communication ability. The largest percentage of students' results on the test was in the very low category of mathematical representation and communication ability. The results showed that students who had good representation and communication abilities would be able to solve problems. There were significant correlations and a very strong correlation between mathematical representation and communication ability with a Pearson Correlation coefficient of 0.915. After obtaining the test result, subjects were selected based on the category of ability to conduct interviews. Based on the results of the interview, the mathematical representation ability that the subject uses well will directly involve mathematical communication skills well, and vice versa.*

Keywords*: mathematical representation, mathematical communication, problem-solving*

Introduction

Problem-solving means engaging in a task for which the solution method is not known in advance. To find a solution, students must draw on their knowledge; through this process, they will often have a new mathematical understanding. Good problems give students the chance to solidify and extend what they know and, when well chosen, can stimulate mathematics learning (National Council of Teachers of Mathematics, 2000). In mathematical learning, problem-solving is a vital thing for students to know. Mathematics does not lose its meaning with mathematical problem-solving because a concept or principle will be meaningful if it can help students achieve optimal results in mastering problem-solving ability (Mataheru, 2019).

In solving a problem, there are three phases, and each phase consists of three aspects. The three phases and their aspects are (1) entry (know, want, introduce), where information enters the mind. (2) Attack (try, maybe, why) in which a phase of questioning the truth and change may occur to solve the problem. (3) Review (check, reflect, and extend), in which examination of the obtained results is carried out. These three phases are related and can be carried out back and forth (Mason, Burton, & Stacey, 2010).

Besides problem-solving, two mathematical abilities are related to each other and affect students' mathematical abilities, namely mathematical representation and communication ability. The ability to represent oneself is a crucial ability that students must develop to succeed in their studies. Mathematical ideas can be represented in a variety of ways, such as figures, concrete materials, tables, graphs, number and letter symbols, spreadsheet displays, and so on. How mathematical ideas are represented is fundamental to how people understand and utilize those ideas. Many of the representations known now that one can take for granted are the result of a process of cultural refinement that took place over many years. When students gain access to mathematical representations and the ideas they express, and when they create representations to capture mathematical concepts or relationships, they acquire a set of tools that significantly expand their capacity to model and interpret physical, social, and mathematical phenomena (NCTM, 2000).

Representation is divided into two parts. External representations are the representations that we can easily communicate to other people, for instance, marks on paper, drawings, geometry sketches, and equations. Internal representations are the images we create in our minds for mathematical objects and processes; these are much harder to describe (Mainali, 2021). Assessment of mathematical representation according to Villegas, Castro, and Gutiérrez (2009) is based on three main external aspects, which include verbal representation of the word problem, pictorial representation, and symbolic representation. Verbal representation of the word problem, consisting fundamentally of the word problem as stated, whether in writing or speaking, Pictorial representations consist of drawings, diagrams, graphs, and any related action. Symbolic representation was made up of numbers, operation and relation signs, algebraic symbols, and any kind of action referring to these.

According to NCTM (2000), mathematical communication is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thoughts to others orally or in writing, they will learn to be clear, convincing, and precise in their use of mathematical language. Asmana (2018) added that written mathematical ability is essential in learning mathematics. Written mathematical communication is the delivery of mathematical ideas by using symbols, figures, vocabulary, and notation without any alteration in written form between two people or more, so the intended meaning can be understood.

Accuracy is indispensable in mathematical communication. Mathematical communication can only be proven accurate if the information submitted is true according to mathematical principles. Furthermore, writing or statement is complete if it is free of flaws. So, it can be concluded that the completeness of written communication in mathematics is information that is presented enough to solve the problem (Dewi, 2014; Asmana, 2018).

There are many materials on mathematical principles that actively involve mathematical representation and communication ability. A linear program is one of the materials learned by the students in the XI grade of senior high school. The purpose of Linear program material is used to find a solution to a problem by creating a mathematical model and finishing it with various mathematical expressions patterns such as graphics, diagrams, figures, or tables. Students struggled and made mistakes when completing linear program problems such as creating a mathematical model, understanding inequality signs, determining the formula, interpreting solutions, and writing calculations. Also, most of the students were not able to create mathematical writing according to what was intended (Feriyanto, 2019; Rahayu & Naila, 2019).

Junita (2016) researched representation and communication ability based on achievement and learning style. It was similar to research conducted by Istiqomah, Tandililing, and Hartoyo (2016) to improve students' communication and representation abilities through the use of Bruner's theory-based worksheets. Several other studies on representation and communication in mathematics have not yet been investigated regarding the relationship between representation and communication in mathematics and the qualitative description of the relationship, although based on the characteristics of representation and mathematical communication, there is a significant relationship. Therefore, this needs to be examined both quantitatively and qualitatively. The relationship between representation and communication ability needs to be examined quantitatively to find out how strong the relationship is so that strengthening one ability will strengthen the other ability. Qualitative research is used to further analyze the indicators of each ability that are interconnected so that the relationship between representation and communication ability can be described qualitatively.

Based on the description that has been presented, this study aims to determine the mathematical representation and communication abilities of class XI students in solving problems in the linear program material, investigate the relationship between representation abilities and mathematical communication abilities, and describe students' mathematical representation and communication abilities.

Method

This research used mixed-methods research with a sequential or gradual strategy, where the researcher first collected quantitative data before analyzing and processing the result, which would be explained later in detail using qualitative research (Creswell, 2013). The sources of the data in this study were 59 years 11 students in 2021/2022 at Senior High School in Ambon, Indonesia. The subjects of the interview were 3 students selected based on their level of ability: S1, who represented the high category; S2, who represented the moderate category; and S3, who represented the low and very low category. Interviews were used to describe mathematical representation and communication for students with high, medium, and low abilities. To collect data, one-time written tests and interviews were carried out, with test instruments and interview guidelines validated by four mathematics education lecturers and one of the teachers at the school used as the research site. The instrument underwent the necessary revisions with input and suggestions from experts until it was declared valid.

In this research, the indicator of representation ability was used according to Villegas et al. (2009), while the indicator of mathematical communication was adopted from the written communication ability indicator according to Asmana (2018), which can be presented in Table 1. This indicator was chosen because it can measure one's ability when representing and eliciting mathematical communication. When students do representation, the characteristics of visual representation, symbols, and verbal representation will appear, as well as the characteristics of written mathematical communication completeness and accuracy which are very important and necessary in solving mathematical problems.

Table 1. Representation ability and communication ability indicators

No	Aspects	Indicators
1	Visual Representation/figure	Creating a figure or graphic to solve the problem given
2	Symbol Representation	Solving the problem by creating a mathematical expression model
3	Verbal Representation	Answering questions by using words or written text
4	Written Communication Accuracy	Writing the rules and calculation steps according to the purpose of the question
5	Completeness of Written Communication	Writing down things that are known and asked, writing down the information of a sketch or graphic made, and drawing the conclusion of a problem

Quantitative data on students' mathematical representation and communication abilities were obtained from criterion-referenced assessment scores according to Ratumanan and Laurens (2015). Meanwhile, qualitative data were obtained from interviews and video recordings of students' work. To determine the relationship between representational ability and mathematical communication, an inferential statistical test was used using SPSS 26.0 software, namely conducting a prerequisite test (data normality criteria) and using the Pearson product-moment test. The correlation coefficient obtained was interpreted according to Sugiyono (2019) to find out whether the relationship between mathematical representation and communication abilities is strong.

Next, qualitative data analysis was carried out to describe the types of representation used by students and compare them with the types of communication that emerged. Qualitative data analysis was carried out based on the steps proposed by Sugiyono (2019), consisting of data reduction, data display, and conclusion drawing (verification). Triangulation is used to ensure the validity of the data by using technical triangulation, namely, comparing the results of students' work with different data collection techniques.

Results and Discussion

At the implementation stage, subjects that represented the categories of high, moderate, low, and very low ability were given a written test of mathematical representation and communication ability. Figure 1 is a description of mathematical representation and communication ability in the problem-solving linear program.

S1 Stood for the Extremely High and High Categories

Figure 1. The figure below shows the results of the representation ability test and the mathematical communication of S1 in solving problems in a linear program. According to the answers provided, the interview was held between a researcher and participant based on problem-solving phases.

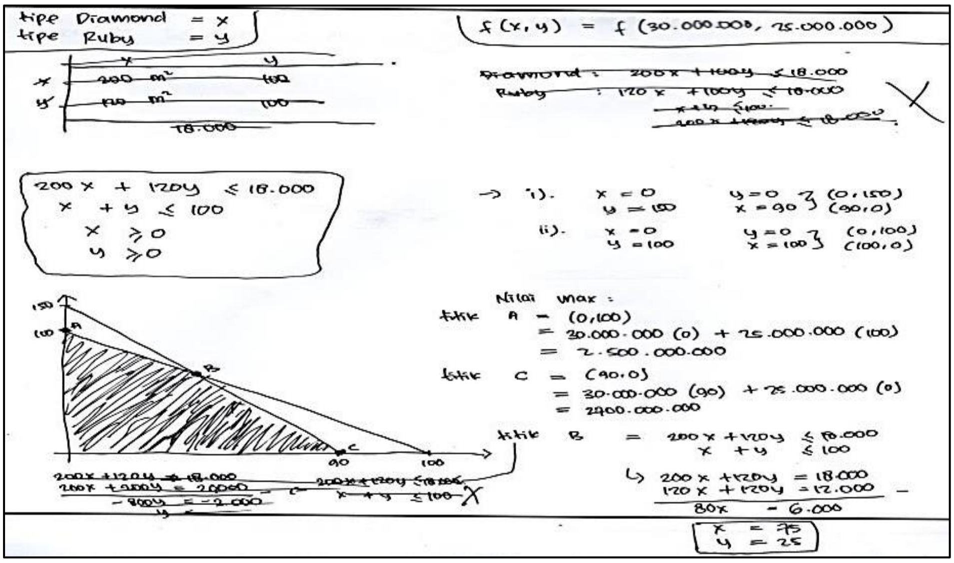

Figure 1. Results of S1 for each phase

The following phases were used to represent and communicate S1 in Linear Program Problem-Solving:

a. The Entry Phase

The interview results of S1 on the *Entry* Phase

R	:	*When reading the question, what things did you think about?*
S1	:	*This question is about determining the maximum profit.*
R	:	*Then what things are known and asked?*
S1	:	*A land area of 18,000 m^2 is known, followed by two types of houses, the diamond (200 m^2) and the ruby (120 m^2), then the objective function, a diamond-type profit of IDR. 30,000,000 and a ruby-type profit of IDR. 25,000,000. After that, I compare the diamond type as x and the ruby type as y, while the question is about the maximum profit obtained from the sale of the two houses.*

R : *Then what strategy was used to solve the question?*
S1 : *First of all, I make the inequality first, draw a graph, and look for the maximum profit.*

According to the answer sheet and interview above, it can be seen that S1 met the indicator of the entry phase, as follows:

1) S1 orally presented what was thought and wanted when reading the questions.
2) S1 presented previously known and requested information in the form of verbal and written representations, such as a diamond-shaped house as type x and a ruby-shaped house as type y.
3) The information was presented in the form of a symbol, such as by creating a function to answer the questions and imagining a diamond-type house and a ruby-type house.
4) The subject was able to explain the problem-solving strategy to determine the question's solution, first by making an inequality, then drawing a graphic, and finding the maximum profit.

b. The Attack Phase

The following is an excerpt from S1's interview on "The Attack Phase".

R : *What steps did you take next to solve the problem?*
S1 : *First of all, I made the inequality, so it's because the land area needed to build a diamond house is 200 m^2, then 200 for the diamond type, and 120 for the ruby type. So* $200x - 120y \leq 18{,}000$ *, why* $\leq 18{,}000$ *. Because* $\geq 18{,}000$ *is an impossible number, and Citraland only owns 18,000 m^2 of land. The second is that the house has no more than 100 units,* $x + y \leq 100$*. Then come* $x \geq 0$ *and* $y \geq 0$*.*
R : *After that?*
S1 : *After making the inequality like this, I make it like the previous question. Make* $x = 0$ *to get the result of* y*, and make* $y = 0$ *to get the result of* x*. Then two points are obtained as well as the second inequality. Get two dots, then pair those dots. After that, look for the set of solutions. The test point that I use here is the same (0,0), but it is smaller (≤) to make it easier. After being tested, it turns out that* $0 \leq 18{,}000$ *means the settlement set area here (while pointing to the graph of the settlement set area). Then, to find the maximum profit, first look for these points, as they have been known: A, C, and D; point C is at (90.0). Then point D is actually (0,0), but there is no profit because (0,0). Then the two points of point B are unknown, so elimination is used between the inequalities* $200x - 120y \leq 18{,}000$ *and* $x + y \leq 100$*. Then I eliminated them, and I got* $x = 75$ *and* $y = 25$*, which means B is at point (75, 25).*
R : *What is next?*
S1 : *Then, when I look for the maximum value of B, the result is IDR. 2,875,000. Furthermore, when I look at points A, C, and B, it turns out that the maximum profit is at point B.*

According to the answer sheet and interview above, it can be seen that S1 met the indicator of the attack phase, as follows:

1) S1 presented the information in the form of a symbol, such as by creating a mathematical model of inequality, then performed the calculations or correctly obtained the solution.

2) S1 carried out the problem-solving plan by presenting known information in the form of communication accuracy, namely, orally or in writing, and by writing the steps of the operation according to the purpose of the questions, such as making a mathematical model of a table, looking for the intersection point of the x and y-axes of the inequality, drawing graphics, and calculating the maximum profit.
3) S1 interpreted every work step and presented it verbally *in* both oral and written ways, such as the starting point, providing graphic descriptions, and concluding with the final answer obtained.
4) S1 presented known information in a visual form, described the solution set area graph, and explained the reasons for determining the intersection point and solution set area.

The visual form in the view of Laamena and Nusantara (2019) is a visual backing that is useful for strengthening students' understanding of the problems they face. Visual representations provide a better figure for students, making it easier to solve problems. Students will construct arguments at each completion step based on visual backing.

c. The Review Phase

The following is an excerpt from the interview with S1 during the *review* phase.

R	:	*What is the conclusion?*
S1	:	*So, the maximum value obtained is at point B (75.25), and the maximum profit obtained is IDR. 2,875,000,000*
R	:	*Do you believe it is necessary to write down the solution to a problem?*
S1	:	*It's important not to get confused like that, because, for example, for people who understand it, it might be easy, but for people who do not understand, they will automatically see the conclusion.*

According to the answer sheet and interview above, it can be seen that S1 met the indicator of the review phase, as follows:

1) S1 did not review the result.
2) S1 performed the problem-solving plan by providing known information in the form of written communication and drawing a conclusion.
3) S1 provided the information using verbal representation, orally and in writing, and explains the process by concluding, and the most important reason to write the conclusion of a question is

S2 Represents the Moderate Category

Figure 2. This is the result of the representation ability test and mathematical communication of S2 in solving problems of the linear program. Based on the answers given by S2, the interview between the researcher and S2 related to the phases of problem-solving.

Mathematical representation and communication of S2 in the problem-solving phases of linear program.

a. Entry Phase

The following is an excerpt from the interview with S2 at the entry phase.

R	:	*After reading the questions, what do you think about the questions?*
S2	:	*After I read the question, I categorized the area of the diamond by the area of the ruby to make the table.*
R	:	*Then what do you know?*
S2	:	*From the question, it can be seen that the area of the diamond type is 200 m^2, and the area of the ruby type is 120 m^2. But in this case, the two types are on a land area of 18,000 m^2, meaning it cannot be more than 18,000 m^2, so it is less than or equal to 18,000 m^2 for the two types of houses. Then what will be known next is the number of houses, because it is not yet known how many types of houses each has, so I categorize the diamond type as x and the ruby type as y, so if that is added, it is less than or equal to 100.*
R	:	*What are the questions asked?*
S2	:	*What was asked was related to the amount of profit. If the answer is known, the type of diamond is valued at IDR. 30,000,000 and the type of ruby is valued at IDR. 25,000,000.*
R	:	*Then, knowing the purpose of the question, what solution or strategy did you think of to solve this problem?*
S2	:	*Because this is asked to make a mathematical model, it's the same as problem number 1. I made the inequality according to the type, drew a graph, and looked for maximum profit.*

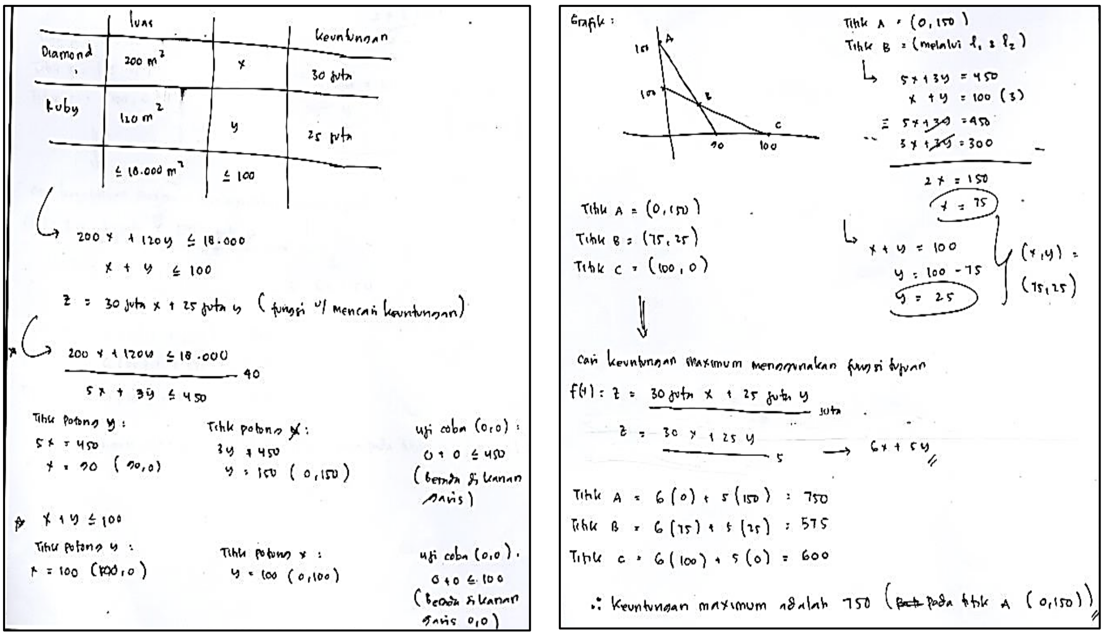

Figure 2. Results of S2 for each phase

According to the answer sheet and interview above, it can be seen that S2 met the indicator of the entry phase, as follows:

1) S2 orally presented the information about what was thought and wanted when reading the questions.
2) By writing every piece of information on the table, S2 presented the known information related to the completeness aspect of written communication.

3) S2 was able to explain the problem-solving strategy to determine the solution to the questions by making an inequality (a mathematical model), drawing a graphic, and finding the maximum profit.

b. The Attack Phase

The following are the results of S2's interview session on the attack phase:

R : *Can you explain the work steps that have been made?*

S2 : *First, I compute the objective function's area, type, and gain inequality. So, because it has been demonstrated that diamond is* x *and ruby is* y*, it means* $200x + 120y \leq 18{,}000$*. Then for the type of house next door, it means* $x + y \leq 100$*. The profit is unknown because I do not know what the final profit from the solution is, so the function is* $30{,}000{,}000x + 25{,}000{,}000y$*. Furthermore, the inequality of area Sakinah reduced to* $200x + 120y \leq 18{,}000$ *divided by 40, so it can function* $5x + 3y \leq 450$*. After that, look for the intersection point again, and if the intersection point is* y*, then* $x = 0$*, and if the intersection point is* x*, then* $y = 0$*, so it will get the intersection point (90, 150), which is for the inequality* $5x + 3y \leq 450$*. Likewise, with* $x + y \leq 100$*, the intersection point will be (100,100).*

R : *Then what about the graph that has been made?*

S2 : *Because the graph already has intersection points, they are included in the graph, and when tested, all the settlement set areas are on the right again because the sign is* $\leq$*, so the set of settlement areas is on the right. Then only remove the outermost points. On the graph, the points are known, namely point A (0.150) and point C (100.0). Then the unknown is the point of intersection between the first line and the second line. how to find the point of intersection of the two lines using elimination. So, returning to the previous inequalities, I eliminated them to get the* x *and* y *values. The first inequality is converted to a meaningful equation:* $5x + 3y = 450$*, and* $x + y = 100$*. so that it can be eliminated, we multiply the second equation by 3.* $5x + 3y = 450$*, and* $3x + 3y = 300$*. If I eliminate the* y *value, then it is finished or zero, so the* x *value to be obtained is 75. Then, from the known* x *value, substitute it into the second equation,* $x + y = 100$*, because* $x = 75$ *and* $y = 25$*. So, I put the intersection point at point B (75, 25), which is the intersection of the first and second lines. Then the outermost points are obtained, namely, A (0.150), B (75.25), and C (100.0).*

R : *After that?*

S2 : *Back to the question. The question is about the maximum profit, and the maximum profit I'm looking for is obtained by plugging the points that have been obtained into the objective function. So, the objective function that you saw earlier is* $z = 30{,}000{,}000x + 25{,}000{,}000y$*, and then I scale it down to get* $z = 6x + 5y$*. From* $6x + 5y$*, I substituted the point values, so I get the benefits. Then I saw the biggest point: point A (0.150) means 6 (0) + 5 (150), which is a profit of 750 million; point B, 6 (75) + 5 (75), is a profit of 575 million; and point C will get 600 million. So, the maximum profit between those three points is 750 million at point A.*

R : *How many types of houses should be built?*

S2 : *For a diamond-type house, there is no value, or 0. Then there are 150 for a ruby-style home.*

R : *Then, in conclusion, here you write down 750; are you sure and don't change it back? Because earlier, when looking for maximum profit, you simplified the objective function from* $z = 30{,}000{,}000x + 25{,}000{,}000y$ *to* $z = 6x + 5y$

S2 : *Actually, I just did not write the millions.*

R : *How many million should it be? 75,000,000 or 7,500,000?*

S2 : *It's still 750 million because I only divided it by eliminating the million. It became $30x + 25y$, then it was simplified again.*
R : *So, the conclusion is written like that?*
S2 : *Yes. In conclusion, the maximum profit is 750 million at point A (0.150).*

According to the answer sheet and interview above, S2 met the indicator of the attack phase, which is as follows:

1) S2 presented the information in the form of a symbol representation, such as making an inequality (mathematical model), then did the calculation with little error in determining the corner point of the solution area.
2) S2 carried out the problem-solving plan by presenting known information in the form of communication accuracy in all and written ways, as well as by writing the steps of the operation according to the purpose of the questions, such as by making a mathematical model of a table, looking for the intersection point of the inequality's x and y-axes, drawing graphics, and determining the maximum profit.
3) S2 interpreted each work step presented verbally, using both oral and written styles such as stating points and concluding the final answer obtained.
4) S2 presented known information in a visual form by describing the solution set of the gap area.
5) S2 followed a problem-solving plan by presenting the known information in the form of written communication and then drawing a conclusion.

S2 used graphics to find the maximum profit. According to Laamena (2019), children with a visual learning style will change problems into visual forms, such as figures, graphs, and others, to make it easier for them to understand the problems presented properly.

c. The Review Phase

Below is the interview between the researcher and S2 during the review phase.

R : *Then, after answering the questions, did you check it again?*
S2 : *I checked again, but I'm sure of the answer.*

According to the answer sheet and interview above, it can be seen that S2 met the indicator of the *review* phase by checking the result again.

S3 Represented the Low and Very Low Categories

Figure 3 is the result of a test for the ability to represent and communicate mathematically in solving linear program problems. Based on the answers given, interviews were conducted between the researcher and the subject according to the problem-solving phases.

a. The Entry Phase

The following is an excerpt from S3's interview on the entry phase.

R : *After reading the questions, what do you think?*
S3 : *What I'm thinking about is "diamond x" and "ruby y" type houses,*

R	:	*What things are known and asked?*
S3	:	*So, what is known is that there are 2 types of houses, namely the diamond type and the ruby type, and 100 units will be built with a diamond-type profit of IDR. 30,000,000.00 and a ruby-type profit of IDR. 25,000,000.00. Then what was asked was the mathematical model and the maximum profit obtained from the sale of the two houses.*
R	:	*Why don't you write down what you know and what you're asked?*
S3	:	*Because it already exists in the question.*

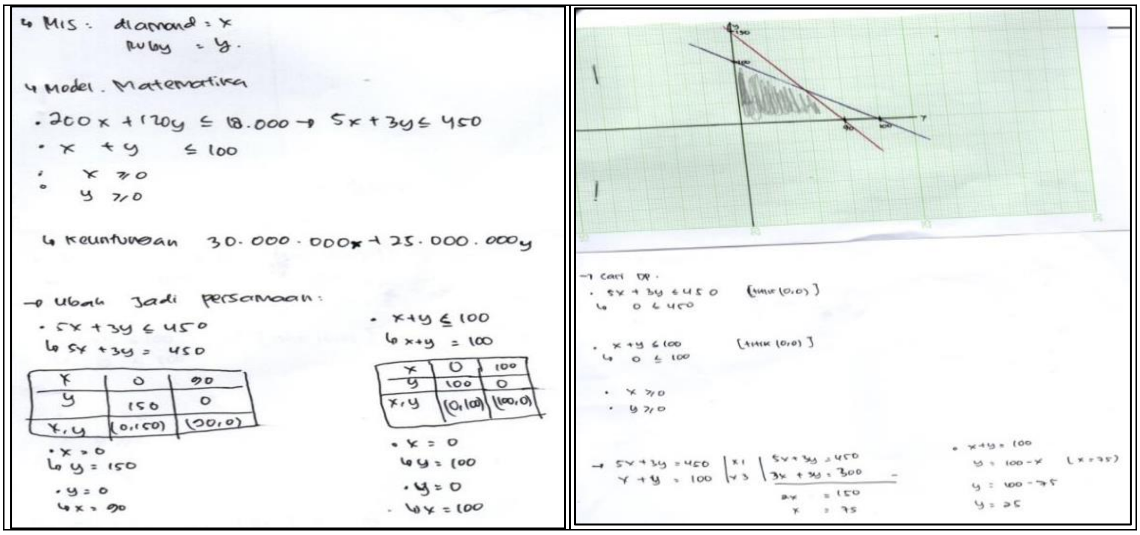

Figure 3. The result of S3 for each phase

According to the answer sheet and interview above, it can be seen that S3 met the indicator of the entry phase, as follows:

1) S3 orally presented the information about what was thought and wanted when reading the questions.
2) S3 did not include known information related to completeness in written communication because S3 thought that it was already available in the question.

b. The Attack Phase

The following is an excerpt from S3's interview on "The Attack Phase".

R	:	*Can you explain the work on question number 2?*
S3	:	*So, the first one is the same as what I said earlier; for example, the diamond is x and the ruby is y. Continue to build a mathematical model. There are $200x + 120y \leq 18{,}000$, so of the total land, 200 and 120 are the land areas required for diamond and ruby-type houses. Then simplify to $5x + 3y \leq 450$, and the second mathematical model is $x + y \leq 100$ for the number of houses for the two, then $x \geq 0$ and $y \geq 0$. Following that, for the 300,000,000 and 25,000,000 diamond-type house profit, which equals $30{,}000{,}000x + 25{,}000{,}000y$,*
R	:	*Why for the mathematical model created which is $200x + 120y \leq 18{,}000$ using the $\leq$ sign? Why not use $\geq$ signs?*
S3	:	*Because here the land area is 18,000m^2, meaning that more or less the total will be 18,000m^2 so I use the $\leq$ sign.*
R	:	*then for $x \geq 0$ and $y \geq 0$, why did Tiara write it?*
S3	:	*That's because there was a problem before, so I wrote again.*

R : *What else is next?*
S3 : *Then just like the previous problem, first convert it into the form of an equation. For the first inequality simplified earlier, it can be* $5x + 3y \leq 450$, *meaning change to the equation* $5x + 3y = 450$. *Then change* x *and* y *to 0, first if change* $x = 0$ *means the value of* $y = 150$ *then the point (0.150), then if* $y = 0$ *means* $x = 90$ *then the point (90.0). Then the second inequality again, change it into the form of an equation that means* $x + y = 100$, *if* $x = 0$ *then* $y = 100$, *likewise if* $y = 0$ *then* $x = 100$ *so get the point (0.100) and (100.0).*
R : *After that, what else?*
S3 : *After that, I look for DHP. Here I tried to use the point (0,0) in the first and second inequalities. If here the* x *and* y *points equal to 0 means* $0 \leq 450$, *then in the inequality* $x + y \leq 100$ *use the point (0,0) obtained* $0 \leq 100$. *Then there are* $x \geq 0$ *and* $y \geq 0$.
R : *Then what else do you do?*
S3 : *Below, it is shown how to find out what the* x *and* y *values are using the substitution elimination method.*
R : *Why do you shade this area?*
S3 : *As before, if the sign is* $\leq$, *then the area is to the left of or beneath the line.*
R : *If that's for* $5x + 3y \leq 450$ *and* $x + y \leq 100$, *then for* $x \geq 0$ *and* $y \geq 0$ *what?*
S3 : *For* $x \geq 0$, *it is the area above the* x*-axis, while* $y \geq 0$ *is to the right of the* y*-axis. then find the settlement area using one of the points on the front. So, this uses the point from the inequality* $5x + 3y \leq 450$ *to find the maximum profit. Then input the objective function, which is* $30{,}000{,}000x + 25{,}000{,}000y$, *to find the maximum profit.*
R : *Why do you only use points (0,150) and (90,0)?*
S3 : *Because at that time I was working on a mandatory math problem, and a teacher said that I only used two DP points, so I only used points (0.150) and (90.0).*
R : *From the graph that you made, it's not the point that is in the solution set area (90,0) and (100,0). Is (0.150) a point that is in the solution area?*
S3 : *The graph depicts a profit-maximizing strategy.*
R : *After that, what is next?*
S3 : *Earlier, I used the points of the inequality* $5x + 3y \leq 450$, *namely (0.150) and (90.0). So, the substitution to the objective function is* $30{,}000{,}000\,x + 25{,}000{,}000$; *for point (0.150), you get 3,750,000,000, while for point (90.0), you get 2,700,000,000. So, why take 3,750,000,000 here? because here the maximum profit is asked for, and it can be seen that the biggest between the two is IDR. 3,750,000,000. Then this is the mathematical model according to what is asked in the problem.*
R : *So, what is the conclusion?*
S3 : *The conclusion is that the maximum profit is IDR. 3,750,000,000.*

According to the answer sheet and interview above, it can be seen that S3 met the indicator of the attack phase, as follows:

1) S3 presented the information in the form of a symbol representation, such as an inequality (mathematical model), then did the calculation with little error in determining the corner point of the solution area.
2) S3 carried out the problem-solving plan by presenting known information in the form of communication accuracy in all and written ways, as well as by writing the steps of the operation according to the question's purpose, such as creating a mathematical model of a

table, locating the intersection point of the inequality's x and y axes, drawing graphics, and determining the maximum profit.

3) S3 interpreted each step of work presented verbally in both oral and written ways, such as the shapes of a house, determined a point, and concluded the final answer obtained.
4) S3 presented known information in a visual form that described graphic areas of the set solution.
5) S3 carried out a plan problem-solving by presenting known information in the form of written communication completeness, such as writing a description of the table and making a conclusion.

c. The Review Phase

The following is an excerpt from S3's interview:

R	:	*When you finished working on question number 2, did you check your work again?*
S3	:	*I checked it, so at that time I immediately drew the graph. Then I read the questions that asked about mathematical models, so I rewrote them at the end. Furthermore, I also provide a box as a conclusion.*
R	:	*So, you've finished getting the maximum profit, then you draw the graph?*
S3	:	*Yes. When I finished work and looked around, maybe a graph was requested, so I drew a graph directly.*
R	:	*Are you sure about the answers you gave?*
S3	:	*Yes, I am*

According to the answer sheet and interview above, it can be seen that S3 met the indicator of the review phase. A subject *check again* results in drawing a graphic and writing another mathematical model under what was questioned. According to Laamena (2019), the process of re-examining produces new arguments reconstructed by students that initially did not appear when answering questions. This is what Mason et al. (2010) meant when he said that the review phase is an important phase to validate student work and allow for improvement in results.

The student's representation ability in the entry phase is based on their initial knowledge. Lestari (2017) stated that initial knowledge affects mathematics learning outcomes. Students with low initial ability are not able to identify all question information well so written communication is incomplete. Verbal and written representations become common patterns that students use in the entry phase. Visual representations in the form of Tables and Graphs as well as symbol representations are used by students to understand problems. These two representations are also used in the attack phase to solve problems. Failure of students to make correct representations will fail in mathematical communication orally or in writing. Students with low ability solve problems with incomplete written communication. The review phase is a stage that is often ignored by students causing the solution obtained to be inappropriate. This can be viewed as a failure of students in rethinking what has been thought (metacognition). According to Laamena and Laurens (2021), Metacognition aspects are important because they relate to one's awareness of the processes and results of thinking. Awareness of errors can occur if students do the review phase correctly.

Based on the test results of 59 students, it was then described in a table presentation so that it was easy to understand. The grouping of students' mathematical representation abilities can be seen in the table below.

Table 2. Grouping of mathematical representation ability test results of students in 2021/2022

Score Interval	Category	Frequency	Presentation (%)
$x \geq 90$	Very High	0	0%
$75 \leq x < 90$	High	4	6.78%
$60 \leq x < 75$	Moderate	1	1.70%
$40 \leq x < 60$	Low	13	22.03%
$x < 40$	Very Low	41	69.49%
Total		59	100%

From Table 2, it was known that four students (6.78%) were in the high category, one student (1.70%) was in the moderate category, thirteen students (22.03%) were in the low category, and forty-one students (69.49%) were in the very low category. Nearly 70% of students' representation abilities are in the very low category. This is because students are not able to use the correct representation according to the problem given. Student representation tends to be imperfect, causing errors in solving the given problem.

In addition to the number of frequencies for the grouping of overall representational ability, a table of average scores can also be made based on the three indicators of mathematical representation, namely pictorial representation, symbolic representation, and verbal representation. The average score

Table 3. Average score of mathematical representation ability: visual, symbolic, and verbal

No.	Representation	Maximum Score	Score	
			$\overline{X}$	%
1.	Visual (pictorial representation)	9	2.95	32.77%
2.	Symbol (symbolic representation)	6	2.36	39.27%
3	Verbal (verbal representation)	9	2.78	30.89%
	Total	24	8,09	

Table 3 shows, that pictorial representation and verbal representation had a maximum score of 9 because they consisted of 3 questions, whereas symbol representation had a maximum score of 6 because it consisted of 2 questions. It can be seen that those three representations had an average score or presentation that was not much different, where the average score of the symbol was higher than the average score of the visual and verbal representations. This indicated that students were able to create a mathematical model and to write the symbols used to complete the mathematical problem.

This table also shows the grouping of the test results for the mathematical communication ability of 59 students.

Table 4. Grouping of mathematical communication ability test results of students in 2021/2022

Score Interval	Category	Frequency	Presentation (%)
$x \geq 90$	Very High	0	0%
$75 \leq x < 90$	High	4	6.78%
$60 \leq x < 75$	Moderate	5	8.47%
$40 \leq x < 60$	Low	10	16.95%
$x < 40$	Very Low	40	67.80%
Total		59	100%

From Table 4. it was known that four students (6.78%) were in the high category, five students (8.47%) were in the moderate category, ten students (6.78%) were in the low category, and forty students (67.80%) were on very low category. Thus, it can be concluded that the biggest test result of the mathematical communication ability of the students was in the very low category. In addition to the total frequency for grouping the total communication ability, a table of average scores can also be made according to the two mathematical written communication namely Accuracy and Completeness. Following is the table of the average score of the two mathematical communication indicators.

Table 5. The average score of mathematical communication ability accuracy and completeness

No.	Communication	Maximum Score	Score	
			$\overline{X}$	%
1.	Accuracy	9	4.37	48.59%
2.	Completeness	9	2.36	26.18%
	Total	18	6.73	

Based on Table 5, it was known that communication indicators, namely accuracy, and completeness, had a maximum score of 9 because it consisted of 3 questions. It can be seen that both indicators of mathematical communication ability have an average score that is quite different, with a difference of 22.41%. The average score of the accuracy communication ability indicator was higher than the average score of the completeness communication ability indicator, which means that students were quite able to solve a mathematical problem according to the rules of correct and precise steps of calculation and operation.

The test results obtained can place students' mathematical representation and communication ability in a very low category. This can be caused by several factors, including: (1) the current learning process is carried out online, so students are not accustomed to or trained to solve questions using representational abilities and good written communication; (2) the duration of the learning process is very short, with limited internet access for students who live far from areas where internet access is available.

The normality test of both mathematical tests was carried out using the Kolmogorov-Smirnor test on SPSS 26.0, which was a prerequisite for hypothesis testing. From the normality test, the results of mathematical representation ability were 0,053, while communication ability was 0,196, implying that the research data collected was normally distributed. To examine the correlation between students mathematical representation and communication ability, the Pearson product moment using SPSS was conducted. The hypotheses tested are as follows.

H_0 : There is no significant correlation between mathematical representation and the communication ability of students

H_A : There is a significant correlation between mathematical representation and the communication ability of students

Hypothesis testing was done for a significance level of 0,05. The results of Pearson product-moment can be seen in Table 6.

Table 6: Test result pearson product moment correlations

		Representation	Communication
Representation	Pearson Correlation	1	.915**
	Sig. (2-tailed)		.000
	N	59	59
Communication	Pearson Correlation	.915**	1
	Sig. (2-tailed)	.000	
	N	59	59

**. Correlation is significant at the 0.01 level (2-tailed).

The -value is 0.000 (0= .05) as shown in Table 6. This means that H_0 was rejected and H_A was. Thus, it can be said that there is a relationship between representational ability and communication ability. Next, mathematical representation and communication ability had a very high and strong positive correlation, r (57) = .915, p = .266. When students' mathematical representation is good, their mathematical communication will also be better, and vice versa.

Based on the report of the test and the interview result, it can be concluded that students in the very high or even high category involving visual, symbolic, and verbal representation ability were capable of solving the problem perfectly. Dahlan and Juandi (2011) stated that the more representation appeared, the easier it got for the students to choose the quickest and most precise way to solve a problem. Students in the moderate category tend to lack proficiency in using representational symbols such as mathematical models. This resulted in the process of solving a problem and acquiring the correct solution becoming hindered. whereas students in the low and very low categories in solving the problem were not able to meet the mathematical representation ability indicator of visual, symbolic, and verbal correctly. This was in line with the statement by Farahhadi and Wardono (2019) that, in designing a mathematical model, there has to be a mathematical representation that communicates certain ideas contained in a problem so that a correct solution to that problem can be acquired. According to Solihah, Hendriana, and Maya (2018), through mathematical communication, ideas can become a goal resulting from reflection, clarification, discussion and development.

In addition, the ability to use the indicators of accuracy and completeness of written communication from each ability category was also different. Subjects in the high ability category were more capable of using both indicators of written communication ability compared to subjects in the moderate and low ability categories, which tended to involve less of an indicator of the completeness of written communication in solving a problem. This was in line with the research conducted by Asmana (2018). It was found that high-ability students' written communication showed accuracy and completeness for every piece of information delivered, whether it was drawing graphics or following calculation rules until the conclusion.

Furthermore, students with moderate and low abilities were accurate with the information conveyed, namely rules and calculations. NCTM (2020) explains the mathematical communication ability of the students reflects the level of understanding of mathematics and the location of the learners' misconceptions. This finding is relevant to Rahmawati, Purwanto, Subanji, Hidayanto, and Anwar (2017) who state that students who have moderate representation skills are better able to represent problems in symbolic form rather than verbally. The completeness of mathematical written communication showed that it was complete for the delivered information things that were known and asked for but incomplete in the calculation and making of drawings and sketches. Sukmaningthias and Hadi (2016) explain that students who have difficulty making representations of problems will have difficulty solving problems. Writing down the questions asked and the information from the graphic to the conclusion was not completed during this research. Ratumanan, Ayal, and Tupamahu (2022) said that low representation ability is caused by Weak prerequisite knowledge and a Weak learning process.

Furthermore, the Pearson correlation coefficient that is similar to both abilities in Table 6 indicates that the correlation that happened between mathematical representation and communication ability was strong and perfect. In line with that, Ariyanti (2016) stated that two mathematical abilities that were correlated and affected students' mathematical abilities were communication and representation ability. Representation itself is one of the keys to mathematical communication ability. Implicitly, this thing indicated that the learning process, which emphasized representation ability, would train students in mathematical communication. Ernaningsih and Wicasari (2017) said, in mathematical communication, students could declare problem solutions by providing precise information on images reinforced by oral communication whereby students can clearly explain each step of the settlement they take. According to Suhaedi (2012), through mathematical communication, students can organize their mathematical thinking processes orally and in writing.

Conclusion

When solving a problem, students with high ability could be able to use their visual, symbolic, and verbal representation ability and could use the accuracy and completeness of written communication appropriately. In contrast, students with moderate or low ability tend to lack the ability to appropriately involve the three representation abilities. Making an error in the creation of a mathematical expression model is an example of symbolic representation. A visual representation such as mistakenly drawing the solution graphically, and a verbal representation such as not writing in words every processing step perfectly. In addition, they were able to meet the indicator of the accuracy of the written communication and tend to be lacking in the completeness of written communication, such as writing things known and asked, writing graphic descriptions, and writing the conclusion.

There is a strong and perfect correlation that occurred between mathematical representation and communication ability. When students can represent problems using visual and verbal representation indicators well, indirect indicators of completeness of written communication will be involved to understand the purpose of the problem, as well as indicator symbols that are interpreted well and clearly will make it easier for students to find solutions to problems using accuracy indicators. In this research, the interview subject was chosen according to the test result and categorization of overall representation and communication ability level, so it was hoped that further research could complement this research by taking the subjects based on each level of mathematical representation and communication ability.

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